Sections 2.1 & 2.2 - I.C.E. Name: KEY

SHOW ALL WORK!!!! ☺

Assume you cannot use a graphing calculator for these problems.
The ones on which you CAN use a graphing calculator are starred.

For questions 1 and 2, determine the left hand and right hand behavior of the graph of function. Fill in the blank with the appropriate sign (positive or negative)

1. $y = 3x^4 - 2x^3 + x$ even, $a > 0$

   as $x \to \infty$, $y \to$ + ∞

   as $x \to -\infty$, $y \to$ + ∞

2. $y = 5x^5 - 6x + 3$ odd, $a > 0$

   as $x \to \infty$, $y \to$ + ∞

   as $x \to -\infty$, $y \to$ − ∞

3. $f(x) = -(x^2 + x - 30)$ (for vertex, use $h = \frac{-b}{2a}$ and $k = f(h)$)

   a) Vertex:

$\frac{-b}{2a} = \frac{-(-1)}{2(-1)} = -\frac{1}{2}$

$f(-\frac{1}{2}) = -(\frac{1}{2}^2 + \frac{1}{2} - 30) = -(-29.25) = 29.25 = \frac{117}{4}$

Vertex: $(-\frac{1}{2}, \frac{117}{4})$

   b) x-intercepts:

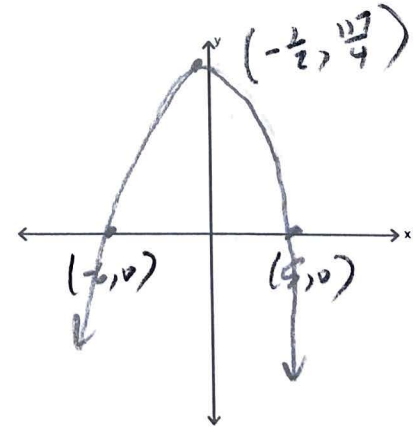
$0 = -(x^2 + x - 30)$

$0 = -(x+6)(x-5)$

$x = -6, x = 5$

$(-6, 0), (5, 0)$

   c)



4. $f(x)=x^2+12x+16$ (find the vertex by completing the square)

a) Vertex:

$=(x^2+12x+36)+16-36$

$=(x+6)^2-20$

vertex $(-6,-20)$

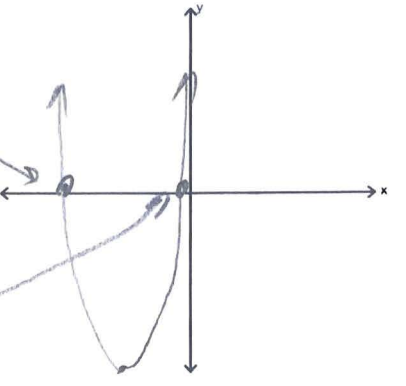
b) x-intercepts:

$x=\frac{-12\pm\sqrt{144-4(1)(16)}}{2}$

$=-6\pm2\sqrt{5}$

$(-6+2\sqrt{5},0),(-6-2\sqrt{5},0)$

c)



For questions 5 and 6, give the equations, in vertex form, for the following graphs.

5. Vertex: $\left(\frac{5}{2},-\frac{3}{4}\right)$, passing through (-2, 4)

$y=a(x-h)^2+k$

$4=a\left(-2-\frac{5}{2}\right)^2-\frac{3}{4}$

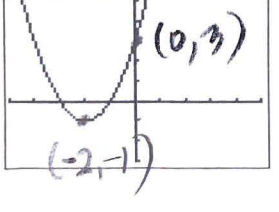
$4=a\left(-\frac{9}{2}\right)^2-\frac{3}{4}$

$\frac{19}{4}=a\left(\frac{81}{4}\right)$

$a=\frac{19}{4}\cdot\frac{4}{81}=\frac{19}{81}$

$y=\frac{19}{81}\left(x-\frac{5}{2}\right)^2-\frac{3}{4}$

6. Use the points that are most clear on this graph:



$y=a(x+2)^2-1$

$3=a(0+2)^2-1$

$a=1$

$y=(x+2)^2-1$

7. $f(x)=x^3-6x^2+9x$

a) find the zeros and determine the multiplicity of each zero

$0 = x(x^2-6x+9) = x(x-3)^2$

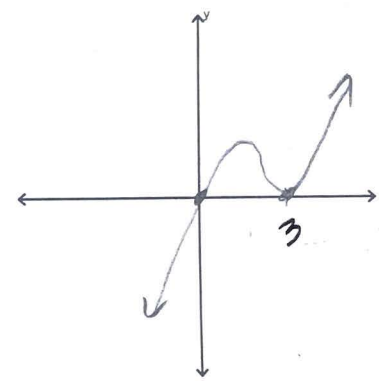
$x=0$ mult 1 (cross), $x=3$ mult 2 (bounce)

b) determine the left/right hand behavior for the polynomial

odd, $a>0$ ↙ ↗

as $x \to -\infty, y \to -\infty$; as $x \to \infty, y \to \infty$

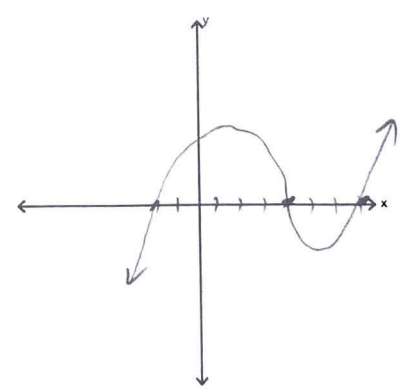
c) use this information to sketch a graph.



8. Find a polynomial of degree n that has the given zeros and then draw a sketch of your polynomial. You may leave it in factored form: you do not need to FOIL it out. (Many correct answers)

Zeros: x = -2, 4, 7 Degree: n = 3

$y=(x+2)(x-4)(x-7)$



what if degree = 4?

$(x+2)^2(x-4)(x-7)$

9. Find a quadratic function **in standard form** whose graph has the given x – intercepts. (Many answers)

x-intercepts: (-2, 0) and (4, 0)

$y = (x+2)(x-4)$ ← factored form

$y = x^2 - 2x - 8$ ← standard form

10. Write a quartic polynomial, $P(x)$, **in standard form** with the following conditions:

Zero at solutions at 1 (multiplicity 2),
Zeros at $\sqrt{7}$ (multiplicity 1) and $-\sqrt{7}$ (multiplicity 1)
y-intercept at (0,21) → solve for "a" → $21 = a(0-7)$, $-3 = a$

$y = a(x-1)^2(x-\sqrt{7})(x+\sqrt{7})$
$y = a(x^2-2x+1)(x^2-7)$
$y = a(x^4-7x^2-2x^3+14x+x^2-7)$
$y = a(x^4-2x^3-6x^2+14x-7)$
$y = -3(x^4-2x^3-6x^2+14x-7)$ $= -3x^4+6x^3+18x^2-42x+21$

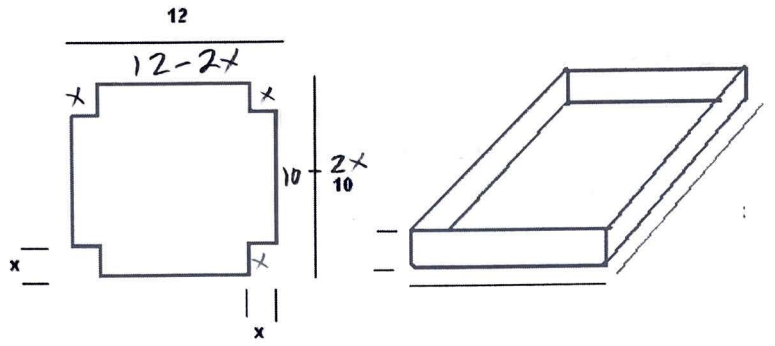
*11. The path of a diver is $y = -\frac{4}{9}x^2 + \frac{24}{9}x + 12$ where y is the height (in feet) and x is the horizontal distance from the end of the diving board (in feet). What is the maximum height of the diver? (Do the work algebraically, then confirm graphically)

$h = \frac{-b}{2a} = \frac{-24/9}{2(-4/9)} = 3$

$k = f(3) = -\frac{4}{9}(3)^2 + \frac{24}{9}(3) + 12 =$ 16 ft ← max height

Confirmed on calculator

*12. An open box with locking tabs is to be made from a square piece of material 12 inches on one side and 10 inches on the other, and this is to be done by cutting equal squares with side length x from the corners and folding up the sides.



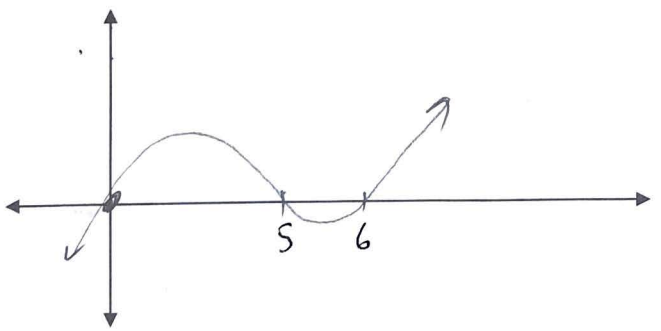
a) What is the volume of the box in terms of x? (you can leave in factored form)

$V(x) = (12-2x)(10-2x)x$

b) What is the domain of the function V?
(in other words, what values of x make this a box that can actually be built?)

$x > 0$ $\quad 12-2x > 0$ $\quad x < 6$ $\quad 10-2x > 0$ $\quad x < 5$ $\quad$ all 3 must be true, so $0 < x < 5$

c) Sketch a graph of the function and find the value of x that will give the maximum volume:
(be sure to adjust your window so you can see the WHOLE graph!)



d) Use your calculator to find the following information:

Max volume will happen at x = 1.811

Maximum volume is 96.77 in^3

*13. Standish throws a softball; the table below shows the height ($h(t)$) of the ball t seconds after it is thrown. Give the quadratic regression equation that best fits the data. Round the coefficients to three decimals.

t	0.35	0.5	1
$h(t)$	8	9.2	7.5

Quadratic model: $h(t) = -17.53846154t^2 + 22.90769231t + 2.130769231$

enter data into L_1, L_2

Stat → calc → quad Reg